HEAT TRANSFER AND THE LEADING CRITICAL POINT OF AN INDESTRUCTIBLE BODY WASHED BY A FLOW OF PARTLY-IONIZED AIR

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The laminar boundary layer at the critical point of an indestructible body of flow of ionized air passing around it is analyzed using refined transfer-coefficient data [1].

1. Presentation of the Problem. For velocities of 12-15 km/sec the incident flow of air passing through the shock wave is heated to a temperature of 12,000-15,000°K. At such high temperatures the air will be completely dissociated into atoms, and some of the atoms will be ionized. When partly-ionized air flows around an indestructible surface a multicomponent laminar boundary layer is formed; this comprises molecules, atoms, ions, and electrons which enter into physicochemical interaction with each other. In addition to this, the high-temperature gas flow will emit strong thermal energy.

In this paper we shall consider the influence of dissociation and ionization on heat transfer, assuming that the boundary layer is optically thin

$$\operatorname{div} \vec{q}_{\mathrm{rad}} = 0.$$

Under this condition, the equilibrium steady-state flow of a multicomponent mixture consisting of nine components (O_2; N_2; NO; O; N; O^+; N^+; NO^+; e^-) undergoing the reactions

$$O_2 \rightleftarrows 2O + Q^*_{O_2},$$
$$N_2 \rightleftarrows 2N + Q^*_{N_2},$$
$$NO \rightleftarrows N + O + Q^*_{NO},$$
$$O \rightleftarrows O^+ + e^- + Q^*_{O},$$
$$N \rightleftarrows N^+ + e^- + Q^*_{N},$$
$$NO \rightleftarrows NO^+ + e^- + Q^*_{NO^+}$$

may be described by the following system of equations [4]:

1) the continuity equation for the mixture of gases

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0; \tag{1.1}$$

2) the diffusion equation for the chemical elements

$$\rho u \frac{\partial \bar{c}_\tau}{\partial x} + \rho v \frac{\partial \bar{c}_\tau}{\partial y} + \frac{\partial \bar{K}_\tau}{\partial y} = 0 \quad (\tau = 0;\ N); \tag{1.2}$$

3) the momentum equation

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right); \tag{1.3}$$

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4) the equation of thermochemical equilibrium, the boundary layer being in an equilibrium state

$$K_{p,i} = \frac{\prod\limits_{j=1}^{v} \bar{c}_j^{\beta_j}}{\prod\limits_{i=1}^{m} \bar{c}_i^{\alpha_i}} (p_e M)^{\Delta n} \quad (i = 1,\ 2,\ \ldots,\ 6); \tag{1.4}$$

5) Dalton's equation

$$\sum_{i=1}^{9} \bar{c}_i M_i = 1; \tag{1.5}$$

6) the energy equation

$$\rho u \frac{\partial I}{\partial x} + \rho v \frac{\partial I}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right] - \frac{\partial}{\partial y}\left(\sum_{i=1}^{9} h_i j_i\right); \tag{1.6}$$

7) the equation of state

$$p = \rho \tilde{R} T \sum_{i=1}^{9} \bar{c}_i. \tag{1.7}$$

In order to close the system of equations we require a relationship between the diffusion fluxes and the concentrations.

The equation of diffusion flux vectors (without allowing for thermal and pressure diffusion) takes the form

$$\frac{\bar{c}_i}{\rho} \sum_{j \neq i}^{9} \frac{\bar{K}_j}{D_{ij}} - \frac{\bar{J}_i}{\rho} \sum_{j \neq i}^{9} \frac{\bar{c}_j}{D_{ij}} = \sum_{j=1}^{9} \left(\frac{\partial \bar{c}_i}{\partial y} \bar{c}_j - \bar{c}_i \frac{\partial \bar{c}_j}{\partial y}\right). \tag{1.8}$$

System (1.8) is complemented by the equation for the sum of the diffusion fluxes

$$\sum_{i=1}^{9} M_i \bar{K}_i = 0. \tag{1.9}$$

2. Method of Effective Gas Coefficients. Even for a small number of components (4-5), the solution of the system of equations (1.1)-(1.9) in general form is quite complicated. An increase in the number of components complicates the problem still further.

However, for the equilibrium case of flow, the system of equations (1.1)-(1.9) may be greatly simplified by using the method of total (effective) coefficients developed by Hirschfelder [5]. The essence of the method is as follows. The thermal flux to the wall may be expressed in the form

$$-q_w = \lambda_w \left(\frac{dT}{dy}\right)_w - \sum_{i=1}^{k} (h_i j_i)_w, \tag{2.1}$$

where the first term allows for heat transfer by conduction and the second by the diffusion of the components.

The expression for the diffusion flux (without allowing for thermal and pressure diffusion) was given by Hirschfelder [5]:

$$\vec{j}_i = \frac{n^2}{\rho} \sum_{j=1}^{k} M_i M_j D_j \cdot \nabla \vec{x}_j. \tag{2.2}$$

In general, the molar concentration is a function of temperature and pressure. Since in the boundary layer (we are considering the neighborhood of the critical point) all the parameters except the pressure ($\partial p/\partial y = 0$) are functions of only one coordinate, the diffusion flux may be expressed in terms of the temperature gradient

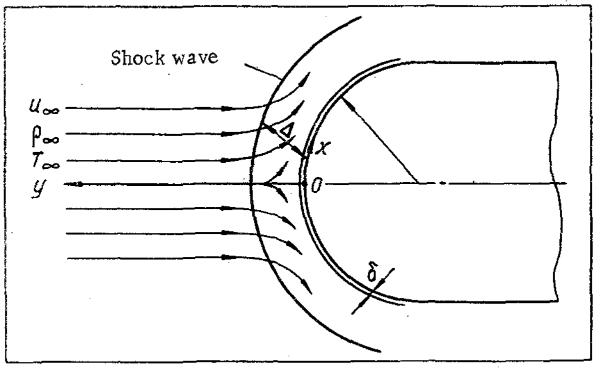

Fig. 1

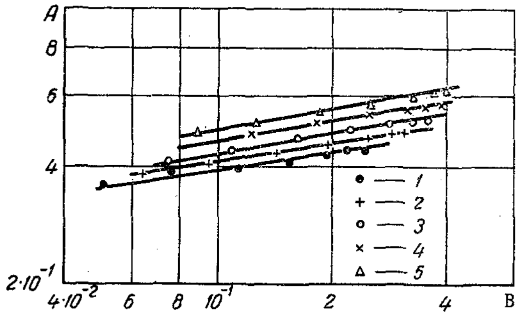

Fig. 2

Fig. 1. Flow of gas around an axially-symmetric body.

Fig. 2. Dependence of the heat-transfer parameter $A \equiv (\mathrm{Nu}/\mathrm{Re}^{0.5}\mathrm{Pr}_{\mathrm{eff}}^{0.4})_{\mathrm{w}}$ on the compressibility $(\mu_e\rho_e/\mu_w\rho_w) \equiv \mathrm{B}$: 1) $T_w = 500$; 2) 1000; 3) 1500; 4) 2000; 5) 2500°K.

$$\vec{j}_i = \left[\frac{n^2}{\rho}\sum_{j=1}^{k} M_i M_j D_j \left(\frac{dx_j}{dT}\right)\right]\cdot\vec{\nabla}T, \tag{2.3}$$

where n is the number of moles in unit volume and D_j is the diffusion coefficient of the multicomponent mixture.

Substituting the expression for the diffusion flux (2.3) into Eq. (2.1) we obtain

$$-q_w = \left[\lambda_w - \sum_{i=1}^{k}\sum_{j=1}^{k}\left(\frac{n^2}{\rho}\right) M_i h_i M_j D_j \left(\frac{dx_j}{dT}\right)_w\right]\left(\frac{dT}{dy}\right)_w. \tag{2.4}$$

The first term in square brackets is the thermal conductivity of the frozen mixture. The second term, associated with dissociation and ionization processes, is called the thermal conductivity of the reacting mixture, or the chemical thermal conductivity

$$\lambda_{\mathrm{chem}} = -\sum_{i=1}^{k}\sum_{j=1}^{k}\left(\frac{n^2}{\rho}\right) h_i M_i M_j D_j \left(\frac{dx_j}{dT}\right). \tag{2.5}$$

The sum of these two coefficients is called the total (effective) thermal conductivity

$$\lambda_{\mathrm{eff}} = \lambda + \lambda_{\mathrm{chem}}. \tag{2.6}$$

Thus the expression for the specific thermal flux to the wall may be written in terms of the effective thermal conductivity

$$-q_w = (\lambda_{\mathrm{eff}})_w \left(\frac{dT}{dy}\right)_w. \tag{2.7}$$

In the same way we may define the total (effective) specific heat of the air at constant pressure

$$c_{p\,\mathrm{eff}} = \left(\frac{dh}{dT}\right)_p. \tag{2.8}$$

Since the total enthalpy of the air, allowing for the energy of formation of its constituents, equals

$$h = \sum_{i=1}^{k} h_i c_i, \tag{2.9}$$

the effective specific heat of the air for p = const takes the form

$$c_{p\,\mathrm{eff}} = \bar{c}_p + \sum_{i=1}^{k} h_i \left(\frac{dc_i}{dT}\right)_p, \tag{2.10}$$

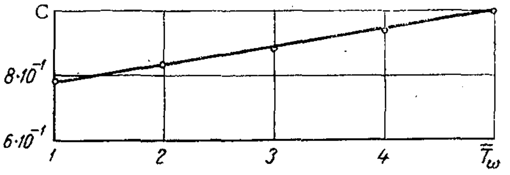

Fig. 3

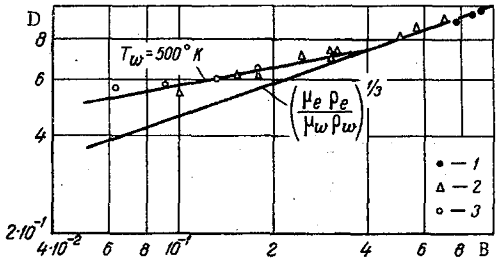

Fig. 4

Fig. 3. Dependence of the heat-transfer parameter $C \equiv \dfrac{\mathrm{Nu}_w}{0.763\,\mathrm{Re}_w^{0.5}(\mathrm{Pr}_{\mathrm{eff}})_w^{0.4}\left(\dfrac{\mu_e\rho_e}{\mu_w\rho_w}\right)^{0.185}}$ on the wall temperature ($\bar{T}_W = T_W/500$).

Fig. 4. Dependence of the heat-transfer parameter $D \equiv \dfrac{\mathrm{Nu}_w}{0.712\left(1+0.07\dfrac{T_w}{T_e}\right)\mathrm{Re}_w^{0.5}(\mathrm{Pr}_{\mathrm{eff}})_w^{0.4}}$ on the compressibility parameter $(\mu_e\rho_e/\mu_W\rho_W) \equiv B$: 1) [7]; 2) [3]; 3) [2].

where $c_p = \sum\limits_{i=1}^{k} c_{p_i}c_i$ is the specific heat of the frozen mixture at p = const. The second term, allowing for the additional heat associated with dissociation and ionization processes, represents the specific heat of the reacting mixture at constant pressure.

3. Transformation of Coordinates. Using the method of total coefficients, we may simplify the system of equations representing the equilibrium boundary layer (1.1)-(1.9) by reducing it to the following:

1) continuity equation for the mixture

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0; \tag{3.1}$$

2) momentum equation

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right); \tag{3.2}$$

3) energy equation

$$\rho u\frac{\partial I}{\partial x} + \rho v\frac{\partial I}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\lambda_{\mathrm{eff}}}{c_{p\mathrm{eff}}}\frac{\partial I}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1-\frac{1}{\mathrm{Pr}_{\mathrm{eff}}}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]. \tag{3.3}$$

Let us consider the flow of gas in the neighborhood of the critical point of an axially-symmetric body (Fig. 1), using the automodel principle, by virtue of which all the quantities depend simply on the transverse coordinate y, apart from u, the value of which is proportional to the coordinate x.

The system of partial differential equations (3.1)-(3.3) may be reduced to a system of ordinary differential equations by using the transformations

$$\xi = \int_0^x \mu_e\rho_e u_e r^2 dx, \qquad \eta = \frac{\rho_e u_e r}{\sqrt{2\xi}}\int_0^y \frac{\rho}{\rho_e}\,dy. \tag{3.4}$$

In the new independent variables (η and ξ) the system (3.1)-(3.3) takes the form:

1) momentum equation

$$(lf'')' + ff'' + \frac{1}{2}\left[\frac{\rho_e}{\rho} - (f')^2\right] = 0; \tag{3.5}$$

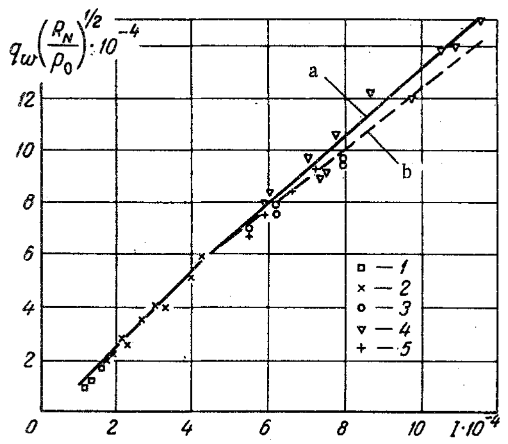

Fig. 5. Dependence of the reduced specific thermal flux $q_w(R_N/P_0)^{1/2}$, kW/m$^{1.5}$ ·bar$^{0.5}$ on the stagnation enthalpy I_0, kJ/kg of the incident flux: 1) [10]; 2) [9]; 3) [11]; 4) [11]; 5) [2]; a) authors' calculation; b) [2].

2) energy equation

$$\left(\frac{l}{\mathrm{Pr}_{\mathrm{eff}}} g'\right)' + fg' = 0. \tag{3.6}$$

The boundary problem has the following boundary conditions:

$$\begin{aligned} &\eta = 0 \quad f_w = 0, \quad f'_w = 0, \quad g = g_w; \\ &\eta \to \infty \quad f'(\infty) \to 1, \quad g(\infty) \to 1. \end{aligned} \tag{3.7}$$

4. Results of the Calculation. The system of equations (3.5)-(3.6) with the boundary conditions (3.7) was solved numerically using the latest data relating to transfer properties [1] and the thermodynamic properties of air [6].

Calculations of the heat transfer from the hot gas to the wall were carried out over the range of variation of stagnation enthalpies corresponding to the ionization region ($I_0 = 40\text{-}115 \cdot 10^3$ kJ/kg). The stagnation pressure and the wall temperature were varied over a wide range ($p_0 = 1\text{-}100$ bar, $T_W = 500\text{-}2500$°K). As a result of each calculation the values of the heat-transfer parameter $(\mathrm{Nu}/\mathrm{Re}^{0.5})_W$ were determined, this parameter being associated with the specific thermal flux by virtue of the equation

$$q_w = \left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_w \left(\frac{I_0 - I_w}{\mathrm{Pr}_{\mathrm{eff},w}}\right) \sqrt{\mu_w \rho_w \left(\frac{du_e}{dx}\right)_0}. \tag{4.1}$$

The results of the calculation are presented in Figs. 2 and 3. Figure 2 shows the logarithmic dependence of the parameter $(\mathrm{Nu}/\mathrm{Re}^{0.5} \cdot \mathrm{Pr}_{\mathrm{eff}}^{0.4})_W$ on the quantity $(\mu_e\rho_e/\mu_W\rho_W)$ for various wall temperatures. We see that in the ionization region the wall temperature has a considerable effect on the heat transfer. In the critical equation for the heat-transfer parameter

$$\left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_w \sim A\,(\mathrm{Pr}_{\mathrm{eff}})_w^{0.4} \left(\frac{\mu_e\rho_e}{\mu_w\rho_w}\right)^b; \tag{4.2}$$

we must therefore introduce the dimensionless wall temperature ($\bar{T}_W = T_W/273$). The curve in Fig. 3 represents the relation between the heat-transfer parameter and the surface temperature. This relationship may be approximated by a linear function to a fair degree of accuracy.

Analysis of the numerical calculations yielded the critical equation

$$\left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_w = 0.763\,(0.75 + 2.73\cdot10^{-2}\bar{T}_w)\,(\mathrm{Pr}_{\mathrm{eff}})_w^{0.4} \left(\frac{\mu_e\rho_e}{\mu_w\rho_w}\right)^{0.185}. \tag{4.3}$$

Figure 4 shows the dependence of the heat-transfer parameter, referred to the heat-transfer parameter for $\mu\rho$ = const, on the dimensionless quantity $(\mu_e\rho_e/\mu_W\rho_W)$, which allows for the compressibility of the gas. The same figure shows some results obtained earlier for the dissociation region [7] and also for the ionization region [2, 3]. We see that in the dissociation region our calculation is excellently represented by the equation

$$\left(\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}\right)_w = 0.712\left(1 + 0.07\frac{T_w}{T_e}\right)\mathrm{Pr}_w^{0.4}\left(\frac{\mu_e\rho_e}{\mu_w\rho_w}\right)^{1/3} \tag{4.4}$$

given in [8]. Extrapolation of curve (4.4) into the ionization region leads to a considerable deviation from the calculated and experimental results obtained for this region. For high temperatures, such as (9-15) 10^3°K, Eq. (4.3) must be used.

Figure 5 shows the dependence of the reduced specific thermal flux on the stagnation enthalpy of the incident flow of air. In the dissociation region the thermal flux was calculated by (4.4)

$$q_w\left(\frac{R_N}{p_0}\right)^{\frac{1}{2}} = 0.712\left(1+0.07\frac{T_w}{T_e}\right)\mathrm{Pr}_w^{-0.6}(\mu_e\rho_e)^{\frac{1}{3}}(\mu_w\rho_w)^{\frac{1}{6}}\times(I_0-I_w)\left(2\frac{p_0}{\rho_0}\right)^{\frac{1}{4}}. \tag{4.5}$$

In this region the theoretical results agree very closely with experiment [9, 10].

In the ionization region the thermal flux was calculated by the critical equation obtained in the present investigation:

$$q_w\left(\frac{R_N}{p_0}\right)^{1/2} = 0.763\,(0.75+2.73\cdot 10^{-2}\overline{T}_w)(\mathrm{Pr}_{\mathrm{eff}})_w^{-0.6}\times(\mu_e\rho_e)^{0,185}(\mu_w\rho_w)^{0.315}(I_0-I_w)\left(2\frac{p_0}{\rho_0}\right)^{1/4}. \tag{4.6}$$

Figure 5 also shows the theoretical results of Khoshizaki [2] as well as experimental data from various authors [2, 11]. Comparison between our own results and those of [2, 3] showed that, despite the considerable difference in transfer coefficients (almost a factor of two), the discrepancy in the specific thermal flux was no greater than 7-10% (Fig. 5); this is very important, since at high temperatures there are no reliable data for the transfer coefficients in the ionization region.

Comparison between the results of the calculation and experimental data [2, 11] obtained in electrical-discharge shock tubes showed (Fig. 5) that the results of the calculation agreed with experiment to an accuracy of 15%.

NOTATION

x, y	are the coordinates;
u, v	are the velocity components, m/sec;
c_i	is the weight proportion of the i-th component;
j_i	is the diffusion flux of the i-th component, kg/m^2 · sec;
ρ	is the density, kg/m^3;
p	is the pressure, bar;
T	is the temperature, °K;
$\overline{c}_\tau = \sum_{i=1}^{k} \frac{n_{\tau,i} c_i}{M_i}$	is the weight proportion of the chemical element;
$\overline{K}_\tau = \sum_{i=1}^{k} n_{\tau,i} \frac{j_i}{M_i}$	is the diffusion flux of the chemical element;
μ	is the viscosity of the mixture, kg/m · sec;
M	is the molecular weight of the mixture;
M_i	is the molecular weight of the i-th component;
D_{ij}	is the coefficient of binary diffusion, m^2/sec;
$h_i = \int_0^T c_{p_i} dT + h_i^*$	is the total enthalpy of i-th component, kJ/kg;
$I = \sum_{i=1}^{k} h_i c_i + \frac{u^2}{2}$	is the total enthalpy of the mixture, kJ/kg;
$\tilde{R}$	is the universal gas constant, J/mole · deg K;
Nu	is the Nusselt number;
Re	is the Reynolds number;
$\mathrm{Pr}_{\mathrm{eff}}$	is the effective Prandtl number;
λ_{eff}	is the effective thermal conductivity, kW/m · deg K;
λ	is the frozen thermal conductivity, kW/m · deg K;
λ_{chem}	is the chemical thermal conductivity, kW/m · deg K;
q_w	is the convective specific thermal flux, kW/m^2;
q_{rad}	is the radiant specific thermal flux, kW/m^2;
R_N	is the radius of nose, m.

Subscripts

w	denotes the wall;
e	denotes the external boundary;

0 denotes the stagnation parameters;
eff denotes the effective (total);
i denotes the component;
τ denotes the element;
$'$ denotes the differentiation with respect to coordinate η.

LITERATURE CITED

1. P'eng-Tsai-Ch'eng and A. L. Pindrokh, Voprosy Raketnoi Tekhniki, No. 12 (1962).
2. Kh. Khoshizaki, Raketnaya Tekhnika, No. 10 (1962).
3. N. B. Cohen, "Boundary-layer similar solutions and correlation equations for laminar heat-transfer distribution in equilibrium air at velocities up to 41,100 feet/sec," Tech. Rept., NASA (1961), NR-118.
4. V. V. Shchennikov, Zh. Vychis. Mash. i Mat. Fiz., 1, No. 5 (1961).
5. J. O. Hirschfelder, in: Problems of the Motion of Long-Range Rocket Heads [Russian translation], IL (1959), pp. 343-364.
6. A. S. Predvoditelev, Tables of the Thermodynamic Properties of Air at 200-20,000°K over the Pressure Range 0.001-1000 Atmospheres, Izd. AN SSSR, Moscow (1959); Izd. AN SSSR, Moscow (1957); Vychis. Tsent., Moscow (1962).
7. G. A. Fay and F. R. Riddell, in: Gas Dynamics and Heat Transfer in the Presence of Chemical Reactions [Russian translation], IL (1962), pp. 190-224.
8. V. S. Avduevskii et al., Fundamentals of Heat Transfer in Aviation and Rocket Technology [in Russian], Oborongiz, Moscow (1960).
9. P. H. Rouse and V. I. Stark, in: Problems of the Motion of Long-Range Rocket Heads [Russian translation], IL (1959), pp. 277-311.
10. Yu. A. Polyakov, in: Physical Gas Dynamics, Heat Transfer, and Thermodynamics of Gases at High Temperatures [in Russian], Izd. AN SSSR, Moscow (1962), pp. 251-260.
11. P. H. Rouse and J. O. Stankevich, Raketnaya Tekhnika i Kosmonavtika, No. 12 (1963); No. 9 (1965).